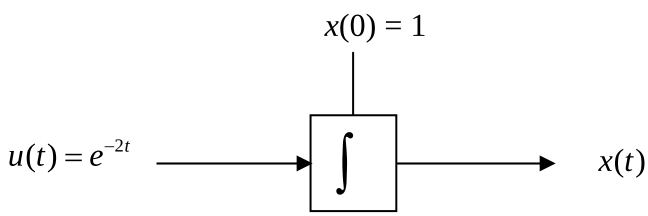
Problem 1 (30 pts)

A continuous integrator with initial condition $x(0)=1$ is shown below.



Fill in the table below where $x(t_n)$ is the exact value of the integrator output at time t_n. Choose $T = 0.05$ for each integrator. Round all answers to 4 places after the decimal point.

n	$t_n = nT$	$x_A(n)$ Explicit Euler	$x_A(n)$ Trapezoidal	$x(t_n)$
0				
1				
2				

Problem 2 (35 pts)

A second order system is modeled by the differential equation

$$\frac{d^2w}{dt^2}+\frac{dw}{dt}+2w=3\frac{d^2u}{dt^2}.$$

The initial conditions $w(0)=\frac{dw}{dt}(0)=0.$

a) Draw a simulation diagram for the system.

b) Find matrices A, B, C, D in the state variable model form y

$$\dot{\underline{x}}=A\underline{x}+Bu$$
$$y=C\underline{x}+Du$$

The single output is $y=w.$

c) The input $u(t)=t,\ t\ge 0.$ Use explicit Euler integration with step size $T=0.1$ to find $y_A(1), y_A(2)$ and $y_A(3).$ Round answers to 4 places after the decimal point.

Hint: First find $\begin{bmatrix} x_{1,A}(1) \\ x_{2,A}(1) \end{bmatrix}, \begin{bmatrix} x_{1,A}(2) \\ x_{2,A}(2) \end{bmatrix}, \begin{bmatrix} x_{1,A}(3) \\ x_{2,A}(3) \end{bmatrix}.$

Problem 3 (35 pts)

An exponential population growth model

$$\frac{dP}{dt} = -kP, \quad (k > 0)$$

is to be simulated in order to approximate the population $P(t)$ for a period of time. The difference equation for $P_A(n)$ intended to approximate $P(t)$ is

$$P_A(n+1) + \alpha P_A(n) = 0$$

a) Find an expression for α in terms of k and the step size T using

   i) Explicit Euler

   ii) Implicit Euler

b) Evaluate α for each integrator when $k = 0.1$ and $T = 1\text{ yr}$. Round answers to 4 places after the decimal point.

c) Fill in the Table below when $P(0) = 5$ million. Round all answers in millions to 4 places after the decimal point.

n	$t_n = nT$	$P_A(n)$, Explicit Euler	$P_A(n)$, Implicit Euler	$P(t_n)$
0	0	5.0000	5.0000	5.0000
1	1			
2	2			

d) The exact solution is given by $P(t) = P(0)e^{-kt}$, $t \ge 0$. Fill in the last column of the table.