Fall 2011 Quiz 1 Solution Name _____

a) The flow out of a tank with cross-sectional area A is given by $f_0 = \alpha H^{1/2}$, $H \geq 0$ where α is a constant and $H = H(t)$ is the tank level at time t. For a tank with $H(0) = H_0$ and no inflow, find the flow out $f_0(t)$. You may use any equations from the text or projects. (5 pts)

From Project 2, Parts d) and e),

$$H(t) = \left[H_0^{1/2} - \frac{\alpha t}{2A} \right]^2$$

$$f_0 = \alpha H^{1/2}$$

$$\Rightarrow \quad f_0 = \alpha \left\{ \left[H_0^{1/2} - \frac{\alpha t}{2A} \right]^2 \right\}^{1/2}$$

$$= \alpha \left(H_0^{1/2} - \frac{\alpha t}{2A} \right), \quad 0 \leq t \leq \frac{2AH_0^{1/2}}{\alpha}$$

b) Sketch the function $f_0(t)$ for the case when $A = 10\ \text{ft}^2$, $H_0 = 9\ \text{ft}$ and and $\alpha = 2\ \text{ft}^3/\text{min per ft}^{1/2}$. (5 pts)

$$f_0(t) = \alpha\left(H_0^{1/2} - \frac{\alpha t}{2A}\right), \qquad 0 \le t \le \frac{2AH_0^{1/2}}{\alpha}$$

$$= 2\left[9^{1/2} - \frac{2t}{2(10)}\right], \qquad 0 \le t \le \frac{2(10)(9^{1/2})}{2}$$

$$= 6 - 0.2t, \qquad 0 \le t \le 30$$