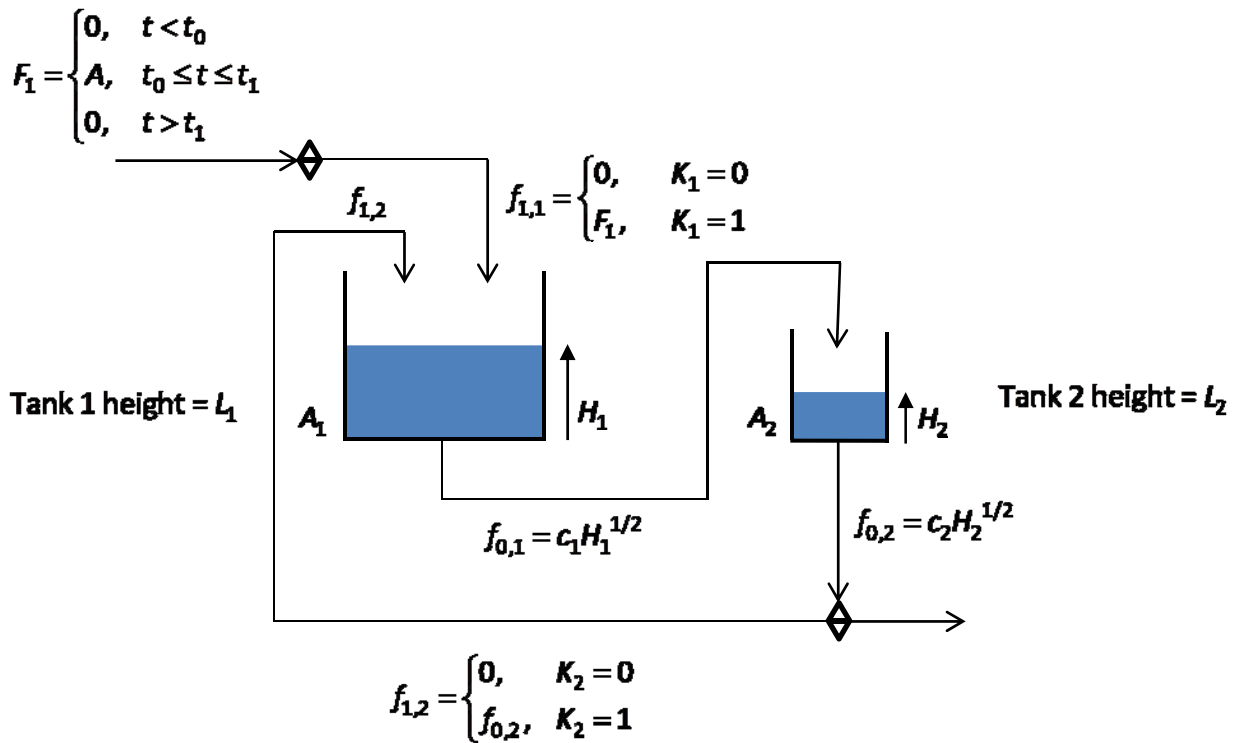


Two tanks are arranged in series as shown.



Baseline parameter values are:

$$A_1 = 4 \text{ ft}^2, \quad A_2 = 4 \text{ ft}^2$$

$$L_1 = 20 \text{ ft}, \quad L_2 = 10 \text{ ft}$$

$$c_1 = 2 \text{ ft}^3 / \text{min per ft}^{1/2}, \quad c_2 = 2 \text{ ft}^3 / \text{min per ft}^{1/2}$$

$$H_1(0) = 0 \text{ ft}, \quad H_2(0) = 0 \text{ ft}$$

$$t_0 = 0 \text{ min}, \quad t_1 = 20 \text{ min}$$

$$A = 15 \text{ ft}^3 / \text{min}$$

$$K_1 = 1, \quad K_2 = 0$$

The overflow from each tank (not shown in figure) is

$$f_{1,s} = \begin{cases} 0, & H_1 < L_1 \\ (f_{1,1} + f_{1,2}) - (f_1)_{\max}, & H_1 = L_1 \end{cases} \quad \text{where } (f_1)_{\max} = c_1 L_1^{1/2}$$

$$f_{2,s} = \begin{cases} 0, & H_2 < L_2 \\ f_{0,1} - (f_2)_{\max}, & H_2 = L_2 \end{cases} \quad \text{where } (f_2)_{\max} = c_2 L_2^{1/2}$$

- 1) Prepare a Simulink diagram for simulating the two-tank system dynamics. Include scopes for viewing the following variables:

$$f_{1,1}, f_{1,2}, f_{0,1}, f_{0,2}, f_{1,s}, f_{2,s}, H_1, H_2$$

In addition, include blocks and scopes for viewing:

- i)  $V_{1,in}$ , the cumulative flow (in cu ft) into Tank 1 from the external source from  $t = 0$  to  $t = t_{final}$ .
- ii)  $V_{1,spill}$ , the cumulative overflow (in cu ft) from Tank 1 from  $t = 0$  to  $t = t_{final}$ .
- iii)  $V_{2,spill}$ , the cumulative overflow (in cu ft) from Tank 2 from  $t = 0$  to  $t = t_{final}$ .
- iv)  $V_{2,out}$ , the cumulative flow (in cu ft) out of Tank 2 not returned to Tank 1 from  $t = 0$  to  $t = t_{final}$ .

Configure the scopes or include "To Workspace" blocks so that the cumulative flows in i) thru iv) are saved in the Matlab Workspace.

- 2) Call the Simulink model from Matlab using the baseline parameter values and plot

- i)  $H_1(t)$  and  $H_2(t)$  on the same graph.
- ii)  $f_{1,1}(t)$ ,  $f_{0,1}(t)$  and  $H_1(t)$  on the same graph.
- iii)  $f_{0,1}(t)$ ,  $f_{0,2}(t)$  and  $H_2(t)$  on the same graph.

Use the 'ode4' integrator with a step size of 0.005 sec. Run the simulation for 30 sec.

- 3) Call the Simulink model from Matlab using the values:

Case I:

$$A_1 = 4 \text{ ft}^2, L_1 = 20 \text{ ft}, c_1 = 2 \text{ ft}^3 / \text{min per ft}^{1/2}, H_1(0) = 0 \text{ ft},$$

$$A_2 = 4 \text{ ft}^2, L_2 = 10 \text{ ft}, c_2 = 2 \text{ ft}^3 / \text{min per ft}^{1/2}, H_2(0) = 0 \text{ ft}$$

$$t_0 = 0 \text{ min}, t_1 = 20 \text{ min}, A = 8 \text{ ft}^3 / \text{min}$$

$$K_1 = 1, K_2 = 0$$

Case II:

$$A_1 = 4 \text{ ft}^2, L_1 = 20 \text{ ft}, c_1 = 2 \text{ ft}^3 / \text{min per ft}^{1/2}, H_1(0) = 0 \text{ ft},$$

$$A_2 = 4 \text{ ft}^2, L_2 = 10 \text{ ft}, c_2 = 2 \text{ ft}^3 / \text{min per ft}^{1/2}, H_2(0) = 0 \text{ ft}$$

$$t_0 = 0 \text{ min}, t_1 = 20 \text{ min}, A = 20 \text{ ft}^3 / \text{min}$$

$$K_1 = 1, K_2 = 0$$

Case III:

$$A_1 = 5 \text{ ft}^2, L_1 = 20 \text{ ft}, c_1 = 2 \text{ ft}^3 / \text{min per ft}^{1/2}, H_1(0) = 10 \text{ ft},$$

$$A_2 = 3 \text{ ft}^2, L_2 = 10 \text{ ft}, c_2 = 2 \text{ ft}^3 / \text{min per ft}^{1/2}, H_2(0) = 5 \text{ ft}$$

$$t_0 = 0 \text{ min}, t_1 = 20 \text{ min}, A = 20 \text{ ft}^3 / \text{min}$$

$$K_1 = 1, K_2 = 0$$

Case IV:

$$A_1 = 4 \text{ ft}^2, L_1 = 20 \text{ ft}, c_1 = 2.25 \text{ ft}^3 / \text{min per ft}^{1/2}, H_1(0) = 0 \text{ ft},$$

$$A_2 = 4 \text{ ft}^2, L_2 = 10 \text{ ft}, c_2 = 1.5 \text{ ft}^3 / \text{min per ft}^{1/2}, H_2(0) = 0 \text{ ft}$$

$$t_0 = 0 \text{ min}, t_1 = 20 \text{ min}, A = 25 \text{ ft}^3 / \text{min}$$

$$K_1 = 1, K_2 = 0$$

Case V:

$$A_1 = 4 \text{ ft}^2, L_1 = 20 \text{ ft}, c_1 = 2 \text{ ft}^3 / \text{min per ft}^{1/2}, H_1(0) = 0 \text{ ft},$$

$$A_2 = 4 \text{ ft}^2, L_2 = 10 \text{ ft}, c_2 = 2 \text{ ft}^3 / \text{min per ft}^{1/2}, H_2(0) = 0 \text{ ft}$$

$$t_0 = 0 \text{ min}, t_1 = 20 \text{ min}, A = 30 \text{ ft}^3 / \text{min}$$

$$K_1 = 1, K_2 = 1$$

For each case, find

- $V_1(0)$ , the initial volume of fluid in Tank 1 (in cu ft).
- $V_2(0)$ , the initial volume of fluid in Tank 2 (in cu ft).
- $V_{1,in}$ , the cumulative flow (in cu ft) into Tank 1 from the external source from  $t = 0$  to  $t = t_{final}$ .
- $V_1(\infty)$ , the final volume of fluid in Tank 1 (in cu ft)
- $V_2(\infty)$ , the final volume of fluid in Tank 2 (in cu ft)
- $V_{2,out}$ , the cumulative flow (in cu ft) out of Tank 2 not returned to Tank 1 from  $t = 0$  to  $t = t_{final}$ .
- $V_{1,spill}$ , the cumulative overflow (in cu ft) from Tank 1 from  $t = 0$  to  $t = t_{final}$ .
- $V_{2,spill}$ , the cumulative overflow (in cu ft) from Tank 1 from  $t = 0$  to  $t = t_{final}$ .

and fill in the table.

