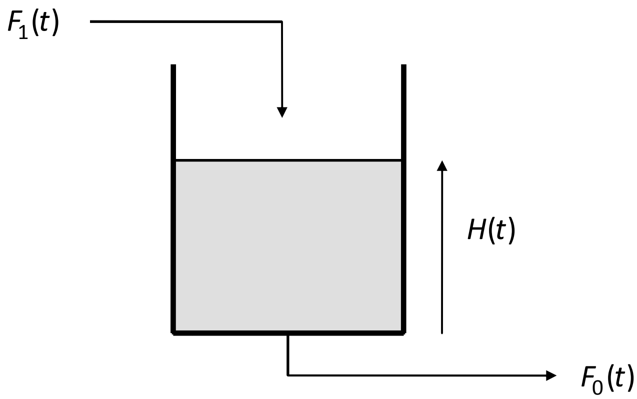
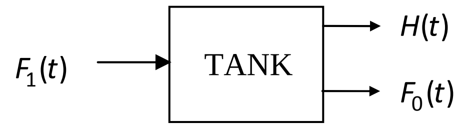
EEL 4890 Project 2 - Tank Dynamics Fall 2011

A single tank with cross-sectional area A ft^2 receives an inflow of $F_1(t)$ in ft^3/min. Outflow $F_0(t)$ is measured in ft^3/min and liquid level in the tank at time t is $H(t)$.





The mathematical model of the tank is

$$A\frac{dH}{dt}+F_0(t)=F_1(t)$$

A simulation of the tank level $H(t)$ and output flow $F_0(t)$ in response to specific input flow profiles $F_1(t)$ and initial tank levels $H(0)$ is required.

a) Assume the outflow from the tank is proportional to the height of fluid in the tank,

$$F_0=\frac{H}{R}$$

where R is the fluid resistance of the tank. Simulate the tank using an explicit Euler integrator with step size T equal to 0.5 min. Use the following baseline values to simulate (generate discrete values) of the tank level and outflow response:

$R=3$ ft/(ft^3/min), $A=10$ ft^2 , $H(0)=$ 0 ft

$F_1(\mathrm{t})=$ 20 ft^3/min, $t\geq 0$

Generate plots of $F_1(t)$, $t\geq 0$, $H_A(n)$, $n=0,5,10,15,\ldots$ and $F_{0,A}(n)$, $n=0,5,10,15,\ldots$ on the same graph for a period of time sufficient to allow the tank to reach steady-state conditions (five time constants). In addition to labeled axes and a title, include a legend with names $F_1(t),\ t \geq 0,\ H_A(n),\ n = 0,5,10,15,...$ and $F_{0,A}(n),\ n = 0,5,10,15,...$.

b) Investigate the sensitivity of liquid level $H(t)$ to variations in the fluid resistance R of the tank. Do this by varying the tank resistance R from 1 to 5 $\text{ft}/(\text{ft}^3/\text{min})$ in increments of 1 $\text{ft}/(\text{ft}^3/\text{min})$. On the same graph, generate the five tank level responses with the value of R shown next to the appropriate response. It is not necessary to graph the simulated responses as discrete data points and the y-axis can be labeled $H(t)$ instead of $H_A(n)$.

c) The analytical solution for the tank level in response to a constant inflow

$$F_1(t) = \bar{F}_1,\ \ t \geq 0$$

is

$$H(t) = R\bar{F} + (H_0 - R\bar{F})e^{-t/AR},\ \ t \geq 0$$

where $H_0 = H(0)$, the initial tank level.

Prepare a graph with the simulated solution $H_A(n)$ using Euler integration $(T = 1)$ and the analytical solution $H(t)$. The simulated solution $H_A(n),\ n = 0,1,2,...$ is to be displayed as blue dots at each discrete point in time connected by a dashed line. The analytical solution $H(t)$ should appear in red as a continuous plot.

Repeat the process for $T = 2.5,\ 5 \text{ and } 10$. Note: There are four graphs, one for each value of T.

Assume baseline conditions except for $H(0) = 25\ \text{ft}$.

d) An alternate model of the tank relates the outflow and liquid level according to

$$F_0 = \alpha H^{1/2}$$

Write a Matlab program which uses the above relationship in conjunction with

$$A\frac{dH}{dt} + F_0 = F_1$$

to simulate the tank dynamics based on forward Euler integration with step size $T = 1\ \text{min}$. Assume $\alpha = 2\ (\text{ft}^3/\text{min})/\text{ft}^{1/2}$ and simulate the filling of an empty tank with cross-sectional area $A = 15\ \text{ft}^2$. Plot the nonlinear tank simulated response $H_A(n),\ n = 0,1,10,20,30,...,600$. Connect the points using a blue dashed line.

e) The analytical solution for the level of a tank with no inflow, i.e. $F_1(t) = 0,\ \ t \geq 0$ and flow out proportional to the square root of the level is given by

$$H(t) = \left[H_0^{1/2} - \frac{\alpha t}{2A} \right]^2$$

For a tank with cross-sectional area $A = 20 \text{ ft}^2$ and $\alpha = 5 \text{ (ft}^3 / \text{min}) / \text{ft}^{1/2}$, use forward Euler integration with step size $T = 0.5 \text{ min}$ to simulate the tank level for the case when $F_1(t) = 0,\ t \geq 0$. Start with an initial level of 25 ft of fluid. Graph the simulated and analytical tank level responses for $0 \leq t \leq 50$ making certain that both responses decrease to zero and remain there. Include a legend with titles $H(t),\ t \geq 0$ and $H_A(n),\ n = 0,1,2,\ldots$.

f) Repeat Part e) by calling the Simulink model from the Matlab script file with the same numerical integrator, step size and final time. Include scopes which display

i) the simulated response

ii) the analytical response

iii) both on the same set of axes

Copy and paste the scope displays into your report.

Compare the results of Part e) and f).