The spread of an epidemic within a population of individuals is under consideration. The disease spreads by contact between healthy (susceptible) people and those inflicted with the disease. People who recover from the disease are immune and no longer susceptible. Those who do not recover die. The following equations apply:

$$\frac{d}{dt}y_1(t) = -ay_1(t)y_2(t) + s(t)$$

$$\frac{d}{dt}y_2(t) = ay_1(t)y_2(t) - by_2(t) - cy_2(t) + d(t) - w(t)$$

$$\frac{d}{dt}y_3(t) = by_2(t)$$

$$\frac{d}{dt}y_4(t) = cy_2(t)$$

$y_1(t)$ = number of susceptible people in the population after *t* days

$y_2(t)$ = number of sick (contagious) people in the population after *t* days

$y_3(t)$ = number of recovered (immune) people in the population after *t* days

$y_4(t)$ = number of people who have died from the disease after *t* days

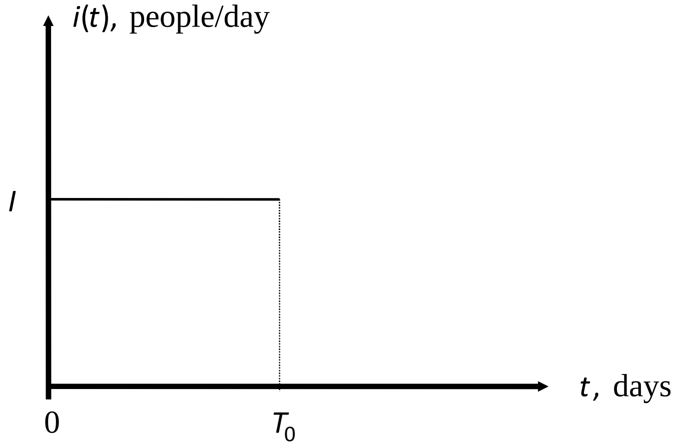
$s(t)$ = arrival rate of susceptible people into the population (people/day)

$d(t)$ = arrival rate of sick people into the population (people/day)

$w(t)$ = removal rate of sick people from the population (people/day)

The constants a, b and c are related to the rate of disease spreading, recovery from the disease, and death from the disease, respectively.

Immigration of people into the population is a constant (I people/day), for a period of T_0 days, after which time the existence of a possible epidemic is realized. Immigration is prohibited thereafter until the disease is under control. Let $i(t)$ represent the immigration of people into the population. Then $i(t)$ can be represented graphically as shown.



Assuming a fixed percentage of immigrants are sick and therefore contagious, i.e.

$$d(t) = \alpha i(t)$$

$$s(t) = (1-\alpha) i(t)$$

where α = fraction of immigrants who are sick.

After the epidemic is recognized, sick people are quarantined (withdrawn) from the population at rate $w(t)$ given by

$$w(t) = \begin{cases} 0 & 0 \le t \le T_0 \\ \beta y_2(t) & t > T_0 \end{cases}$$

where β = fraction of sick people isolated from the population.

The total number of people in isolation after t days, $q(t)$ is the accumulation of people who have been removed. Mathematically, this can be expressed as

$$q(t) = \int_0^t w(t')dt'$$

Baseline numerical values of the system parameters are:

a = 0.00001 (people-day)$^{-1}$

b = 0.5 people/day per person

c = 0.05 people/day per person

α = 0.1

β = 0.1(10%) per day

T_0 = 10 days

I = 200 people/day

Assume the initial population consists of 98,000 susceptible people with no sick individuals present.

a) Prepare a SIMULINK block diagram to model the system. Set the numerical values of the system parameters and initial conditions in a Matlab script file which calls the Simulink model. (Remember to set the initial condition on the y_1 integrator to 98,000 in the script file.)

   Simulate the transient response of the system for 50 days using the fixed step ode4 integrator with step size $T = 0.025$ days.

b) Configure the scopes to save $y_1(t), y_2(t), y_3(t), y_4(t)$ and plot the time histories in Matlab on a single graph.

c) After t days, the total number of people alive (and not quarantined) in the population, $y(t)$ is given by

$$y(t) = y_1(t) + y_2(t) + y_3(t)$$

   In Matlab, plot $y(t)$ vs t.

d) Configure the Simulink scope for $q(t)$ to save the values and use Matlab to plot $q(t)$ vs t.

e) Plot the quantity $P(t) = y_1(t) + y_2(t) + y_3(t) + y_4(t) + q(t)$ vs t and comment on the meaning of $P(t)$ and the resulting plot.

f) Perform a sensitivity analysis of $y_4(t)$ as β varies from 0 to 0.25 in steps of 0.05. Plot separate curves of $y_4(t)$ vs t for each value of β on the same graph. Comment on the results.