Informed search for plans

Some figures are derived from the slides created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley, available at http://ai.berkeley.edu.

Goal information

- The algorithms we discussed until now (depth first, breadth first, uniform cost search) assumed that the only information we have about the goal is a binary $G(\cdot)$ function
    - that is, we will recognize the goal, when we found it.

Goal information (cont'd)

- How do we go from Orlando to San Francisco?
  - It is to the west from us.
  - So we probably have to go mostly to west
  - But taking every action "to west" does not take you there
- In practice, we might have more information about the goal
  - But this information can be vague, incomplete, uncertain, probabilistic or wrong
- **Challenge:** how do we integrate additional information about the goal into our search for a plan.

Heuristics

- A function $h(s) \rightarrow \mathbb{R}$ that estimates how far is a state from the goal
- It is a way to encapsulate knowledge about the goal
- We can assume that if $G(s) = true$ then $h(s) = 0$

Examples of heuristics

- The "as-the-crow-flies" distance to San Francisco
- The number of horcruxes remaining
- The number of items remaining on the original side of the river

In general, we use the term **heuristics** when we are talking about a solution that is not formal, but it relies on trial and error, or some kind of human knowledge or intuition. It is the same greek root as Archimedes' heureka!

Greedy search

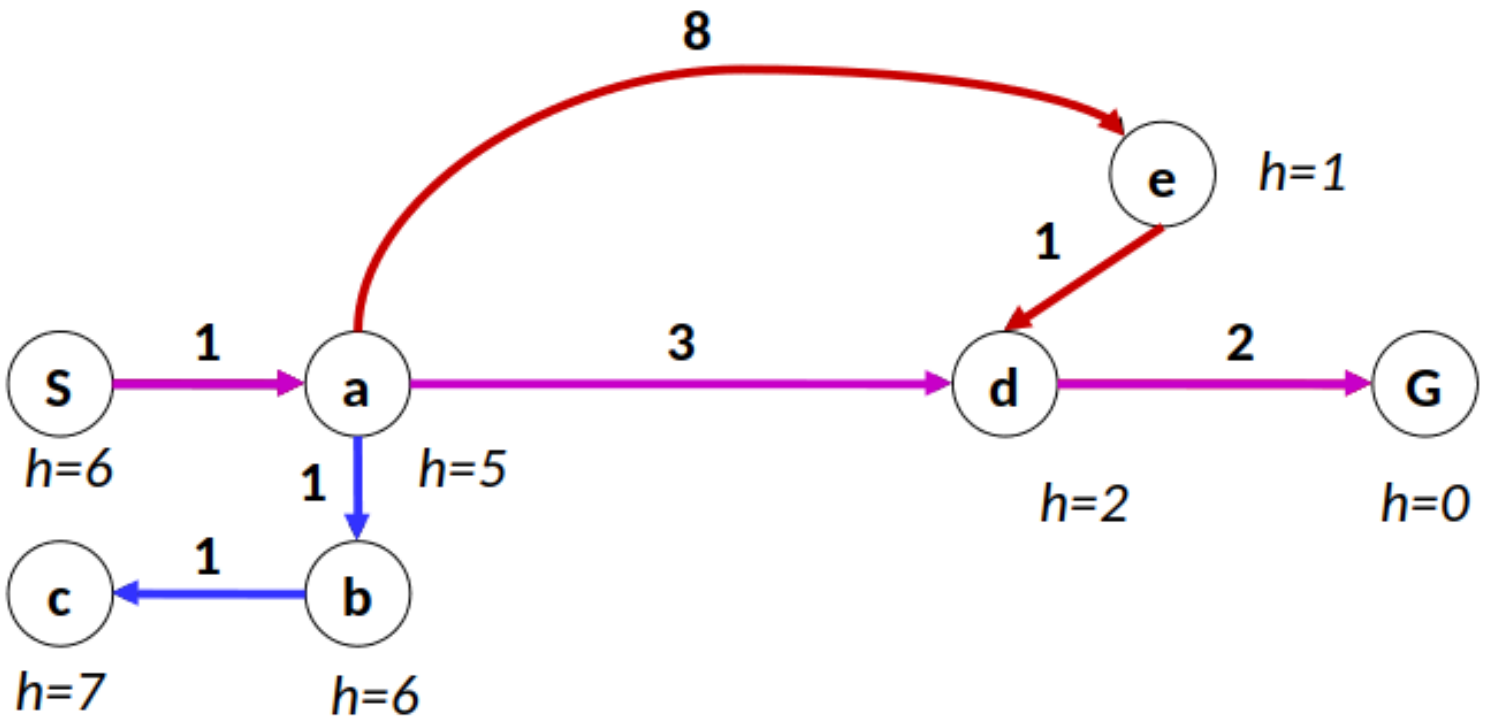
- Strategy: expand the node with the lowest heuristic value
  - Make the fringe a priority queue ordered by $h(n)$
  - Where h(n) is the heuristic of the state marking the node
  - Pick the smallest
- Sometimes also called as **best-first search**
- How good it is? Depends on the quality of the heuristics:
  - If the heuristics gets the *ordering* right (not necessarily the *values*) - you go straight to the solution!
  - If the heuristic is wrong, you can end up like in DFS, or worse
- The quality of the heuristics reflects our understanding of the problem.

Properties of greedy search

- Optimal: no, the heuristics might lead you on a non-optimal path or to the non-optimal goal.
- Space and time complexity: can range anything between BFS and DFS.
  - If the heuristics is provably good, it can be better than any of those.
  - Insight: DFS and BFS are heuristic search with a particular type of heuristic
- Can you get **stuck**?
  - No, if you are following the standard tree search algorithm - you try best first, but will explore the other ones later.
  - But you can end up endlessly deep, like in DFS.

A* search

- Combines Uniform Cost Search and Greedy
  - Uniform cost orders by path cost $g(n)$ aka *backward cost*
  - Greedy orders by estimated goal proximity $h(n)$ aka *estimated forward cost*
  - A* orders by sum $f(n) = g(n) + h(n)$


8
e
h=1
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2
G
h=6
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h=2
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c
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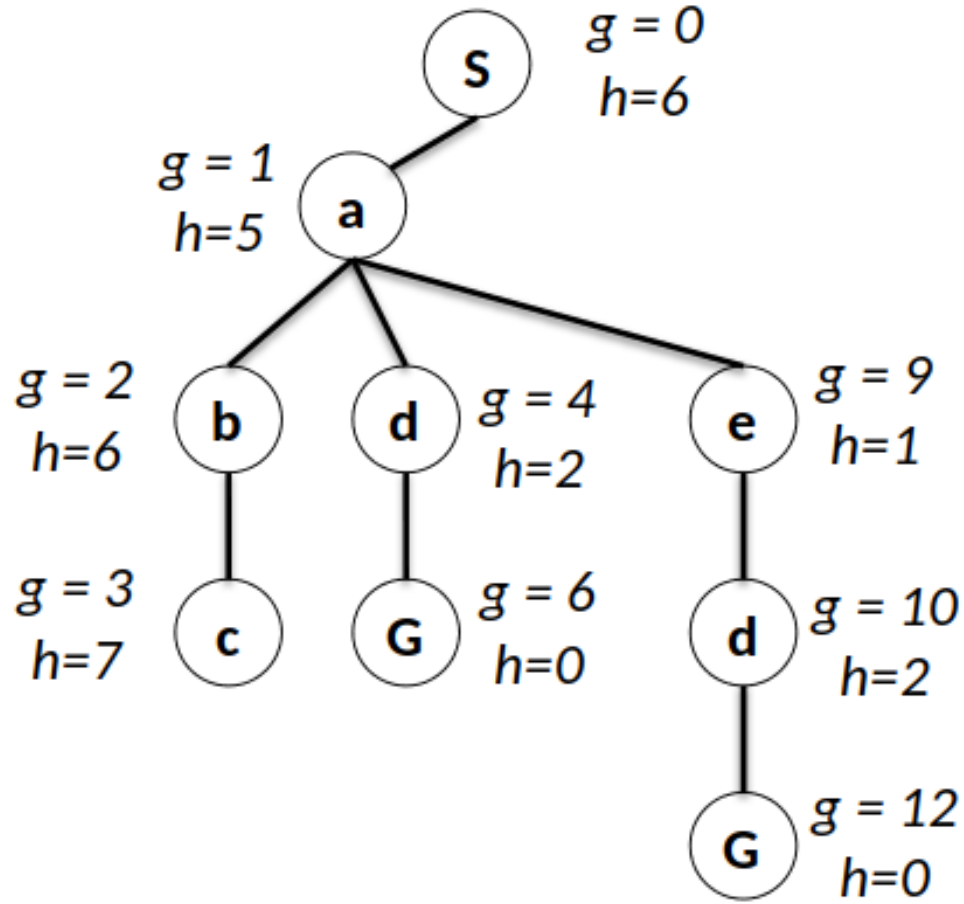
b
h=7
h=6

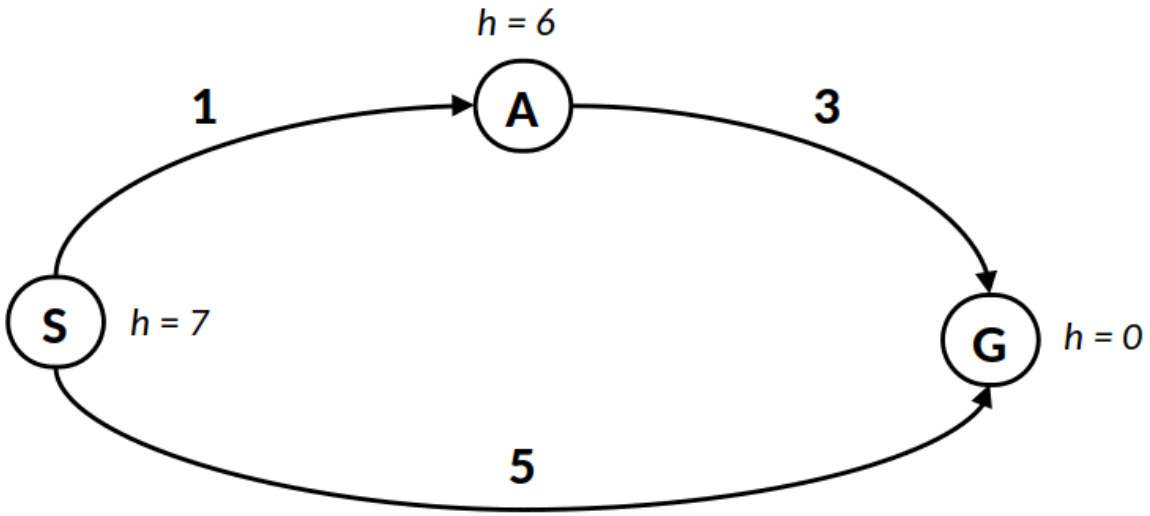

S
g = 0
h=6
a
g = 1
h=5
b
g = 2
h=6
d
g = 4
h=2
e
g = 9
h=1
c
g = 3
h=7
G
g = 6
h=0
d
g = 10
h=2
G
g = 12
h=0

When should A* terminate

- Don't stop when we add the goal to the fringe!
  - The fringe is not FIFO - it is possible that the goal we added is not the one that will come out first!
- Only stop when we take out a node labeled with a goal from the fringe

Is A* optimal?

- Not in this case!
- The heuristic misled us!
- But if we need a perfect heuristic, why do we bother with A*
- Turns out we don't need the heuristic to be perfect, we only need it to be **optimistic**



Admissible heuristics

- Inadmissible (pessimistic) heuristics break optimality by trapping good plans far down on the fringe
- Admissible (optimistic) heuristics never overweigh true costs:

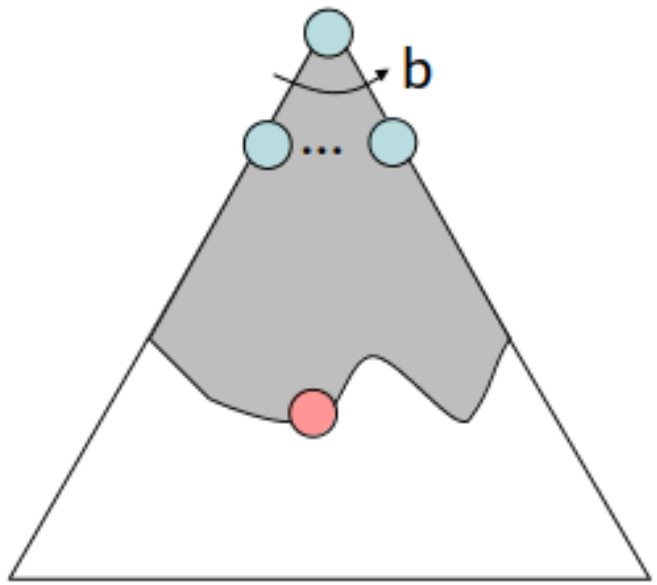
$$0 \leq h(n) \leq h^*(n)$$

- where $h^*(n)$ is the true cost to a nearest goal.

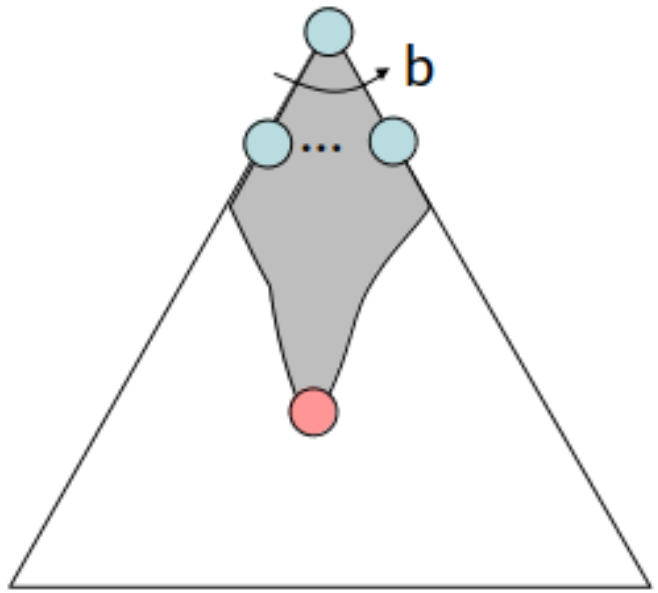
A* properties

- Uniform cost expands equally in all directions
- Greedy expands sharply towards what it thinks is the goal
- A* expands mainly towards the goal but also other directions

Uniform-Cost



A*



Creating admissible heuristics

- The critical challenge in making A* work for you is to come up with a good admissible heuristic
- Trivial admissible heuristic: $h(n) = 0$
  - Reverts A* to uniform cost search
- Perfect heuristic $h(n) = h^*(n)$
  - Go straight to the goal
- There is a partial ordering between admissible heuristics (*dominance*)
- The max of admissible heuristics is admissible

$$h(n) = max(h_a(n), h_b(n))$$

Admisible heuristics from relaxed problems

- How do we get good admissible heuristics?
- One way: try to solve a **relaxed problem**
  - A problem which is in some way easier than the original one
- One easy way to create a relaxed problem: add new actions
  - Imagine that the agent is a superhero!!!
  - Eg. ability to fly - Euclidean distance
  - Eg. ability to pass through walls - Manhattan distance
  - Eg. ability to destroy horcruxes from distance - horcrux count

Extra work in tree search

- Until now, all the algorithms were variations of **tree search**
  - You can have many plans in the tree labeled with the same node
  - Can lead to (exponentially more) extra work

Graph search

- Idea: never **expand** a state twice
- Augment the tree search algorithm with a **closed set** the set of expanded states
- Before expanding a node, check if the state was expanded before
  - Yes: skip it
  - No: expand it and add it to the closed set
- The closed set only used for membership check: implement as a hashset.

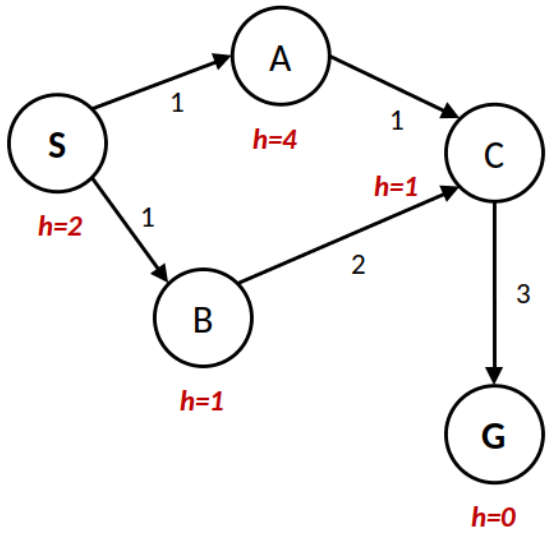
Graph search properties

- Any tree search algorithms can be converted to graph search
- Graph search obviously avoids some expansions
- Does it change the properties?
  - Space complexity: increased, due to the closed set
  - Completeness: whatever states had been expanded before, they will be expanded now as well, so the algorithm retains completeness
  - Optimality?

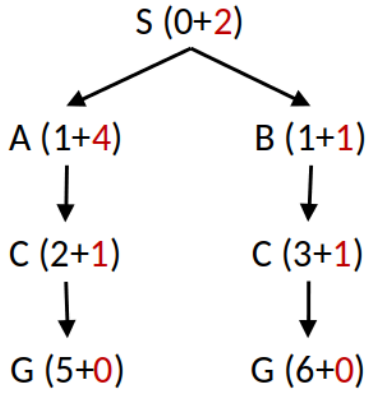
A* graph search optimality

- Admissible heuristic not sufficient
- Heuristics also needs to be **consistent**

State space graph



Search tree



Consistent heuristics

- Admissibility: heuristic cost $\leq$ actual cost to goal
  $h(A) \leq$ actual cost from A to G
- Consistence: heuristic "arc" cost $\leq$ actual cost for each arc
  $h(A) - h(C) \leq$ cost(A to C)
- Consequences of consistency:
  - The f value along a path never decreases
  - A* graph search is optimal
- How do we find consistent heuristics?
  - Relaxed problems will be consistent!!!

Overall perspective on A*

- A* is the perfect algorithm for combining systematic search with background knowledge and intuition
- Very extensive set of applications
  - Pathing, routing problems
  - Resource planning problems
  - Video games
  - Robot motion planning

Limits on A*

- A* does not handle well:
  - Replanning
    - There several extensions that try to avoid recalculating from scratch
  - Planning in very large (or even infinite) state spaces
    - Probabilistic algorithms, such as Rapidly Exploring Random Trees - RRT
- A* had also been previously also used for
  - Language analysis
  - Machine translation
  - Speech recognition
  - As of 2024, these problems are usually treated with different approaches.

State of the art in planning

- Planning is a significant problem in business, manufacturing, transportation etc.
- There is a significant subfield of computer science that deals with finding specialized planning solutions.
- Usually, it deals with a deeper analysis of the effect of actions (eg. pre-conditions, post-conditions)
- Research community centered on International Conference on Automated Planning and Scheduling (ICAPS).
    - Hosts the International Planning Competitions, ongoing efforts to find the best algorithms